PARAMETERS OF BUBBLES AND DROPS MOVING IN A LIQUID

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The author uses wave theory to examine the processes of formation of bubbles and drops moving in a liquid.

Bubbles and drops, ascending and descending in a liquid, and depending on the properties of the continuum and discrete phases, as their size increases, change their shape from spherical to ellipsoidal, and to a shape which we shall call cup-shaped. During this process their eccentricity increases. When they reach drops and bubbles of a certain size subdivision occurs.

According to [1], the discrete phase (bubbles and drops) and the continuum phase can be regarded as a wave of wavelength $\lambda_e = \pi D_e$, moving with a velocity [2]

$$u_\lambda = [2\sigma/(\rho_1 + \rho_2) D_e + g D_e (\rho_1 - \rho_2)/2(\rho_1 + \rho_2)]^{1/2}. \tag{1}$$

For small enough Re the actual speed of motion of the bubble (or drop) is limited by the possible flow speed of the liquid relative to a moving body defined by viscous forces, and being [3]

$$u_\eta = 4\Delta\rho g D_e^2/3c_f\eta_1, \tag{2}$$

where $c_f = k_f/\mathrm{Re}$.

With increase of the size of the moving body u_η increases and when it reaches $u_\eta \geq u_\lambda$, since the possible speed of motion of the liquid exceeds the speed of motion of the wave, because of their relative motion in the bubble (or drop) circulation arises which leads to breakdown of the spherical shape. Equating Eqs. (1) and (2) and using a transformation, we obtain the condition for deformation of a sphere

$$\mathrm{Bo} \geqslant (3)^{4/5} k_f^{4/5} [2 + \mathrm{Bo}/2]^{2/5}/2^{8/5} (\mathrm{Ka})_1^{1/5}, \tag{3}$$

where $(\mathrm{Ka})_1$ is the modified Kapitza number, first used by Kapitza to describe the properties of a liquid [4] (the criterion is known abroad as the Morton number).

The solution of this case, presented in [3], is analogous in the composition of the parameters, but evidently less accurate.

In the motion of a bubble (or a drop) in liquids of high viscosity loss of sphericity can occur at Bo values smaller than as computed by Eq. (3). When the condition $\pi D_e \geq \pi_{cr}$ holds, i.e., when the wavelength of the bubble (or drop) becomes larger than critical, the wave ceases to be stable, which leads to its growing, i.e., a change of shape of the bubble (or drop). According to [2], $\lambda_{cr} = 2\pi(\sigma/\Delta\rho g)^{1/2}$, and, consequently, this occurs for $D_e \geq 2(\sigma/\Delta\rho g)^{1/2}$. In [5], which is usually cited in these investigations, the authors reached this result on the basis of an arbitrary qualitative assumption without observing the quantitative side of the latter. Thus, for small enough values of Ka_1 the loss of sphericity by a moving bubble or drop will occur when

$$\mathrm{Bo} \geqslant 4. \tag{4}$$

If we assume that the moving body acquires the shape of a symmetric ellipsoid of revolution, its eccentricity can be determined from the following considerations.

From the forward point of the moving body on the side of its axis capillary waves of length $\lambda_\sigma = 2\pi(\sigma/\Delta\rho g)^{1/2}$ expand with speed u_σ, while the body itself moves with speed u_λ like a gravity wave. Here the frontal surface of the body is curved so as to achieve a con-

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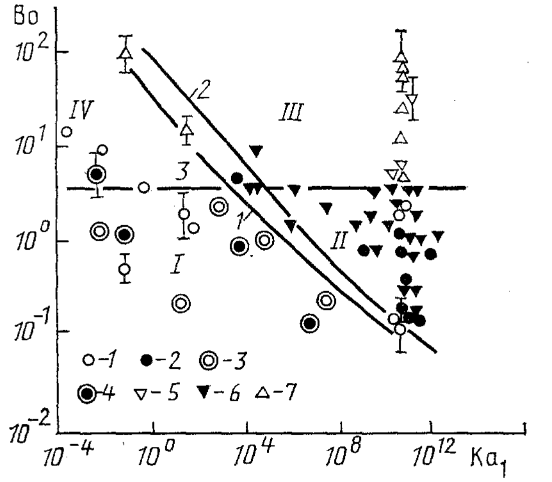

Fig. 1. Conditions for change of shape of bubbles and drops: 1, 2) deformation of spherical bubbles and drops, respectively; 3, 4) the appearance of circulation flows in bubbles and drops; 5, 6) transitional shape of bubbles and drops; 7) cup-shaped bubbles.

stant dwell time of the capillary wave at the interface with the medium. Therefore, in a time in which the capillary wave goes a distance equal to the semiaxis of the ellipsoid a_0, the gravity wave must traverse a distance equal to the semiaxis b_0.

According to [2], the speed of motion of the capillary wave in the conditions examined is

$$u_\sigma = u_{\mathrm{fr}} + (g\delta)^{1/2}[(\rho_1 - \rho_2)/(\rho_1 + \rho_2)]^{1/2}. \tag{5}$$

For large enough bubble (or drop) size the speed of the liquid at the boundary u_{fr} is close to the speed of motion u_λ [6].

The layer thickness δ is a maximum on the short axis of the ellipsoid, and is $\delta = 2b_0$. As the wave moves toward the equator of the ellipsoid the value of δ decreases, and on the equator $\delta = 0$ and $u_\sigma = u_{\mathrm{fr}}$, i.e., formation of the bubble (or drop) surface ends.

Substituting $u_0 \equiv \partial a/\partial\tau$ and $\delta = 2(b_0 - u_\lambda\tau)$ into Eq. (5) and integrating the equation in the limits $a = 0$ for $\tau = 0$ and $a = a_0$ for $\tau = \tau_0$ under the postulate that $u_{\mathrm{fr}} = \mathrm{const}$, we obtain

$$a_0 = u_{\mathrm{fr}}\tau_0 + 2^{3/2}g^{1/2}b_0^{3/2}(\rho_1 - \rho_2)^{1/2}/3u_\lambda(\rho_1 + \rho_2)^{1/2}. \tag{6}$$

Since $e = a_0/b_0$ for a symmetric ellipsoid of revolution, dividing the left and right sides of Eq. (6) by b_0, taking into account that $u_\lambda = b_0/\tau_0$, and substituting Eq. (1) into the second term on the right of Eq. (6), after transformations we obtain an expression for the eccentricity e in implicit form:

$$e = u_{\mathrm{fr}}/u_\lambda + 2/3e^{1/3}[1/2 + 2/\mathrm{Bo}]^{1/2}. \tag{7}$$

For a cup-shaped bubble in Eq. (5) we have $\delta = b_0 - u_\lambda\tau$, $a_0 = R\sin\theta$, $b_0 = R(1 - \cos\theta)$, $R = D_e/2^{1/3}(\cos^3\theta - 3\cos\theta + 2)^{1/3}$. Using these relations and integrating Eq. (5) over the same limits as for the ellipsoidal body, after transformations we write an equation for the angle θ in implicit form

$$\sin\theta = (1 - \cos\theta) + (2)^{5/6}(1 - \cos\theta)^{3/2}/3(\cos^3\theta - 3\cos\theta + 2)^{1/6}[1/2 + 2/\mathrm{Bo}]^{1/2}. \tag{8}$$

When they reach specific sizes the bubbles and drops may break up. Using the laws for subdivision of a body in the flow as proposed in [7] we can predict that breakup occurs when two conditions hold: The minimum wavelength arising in flow over the phase interface and

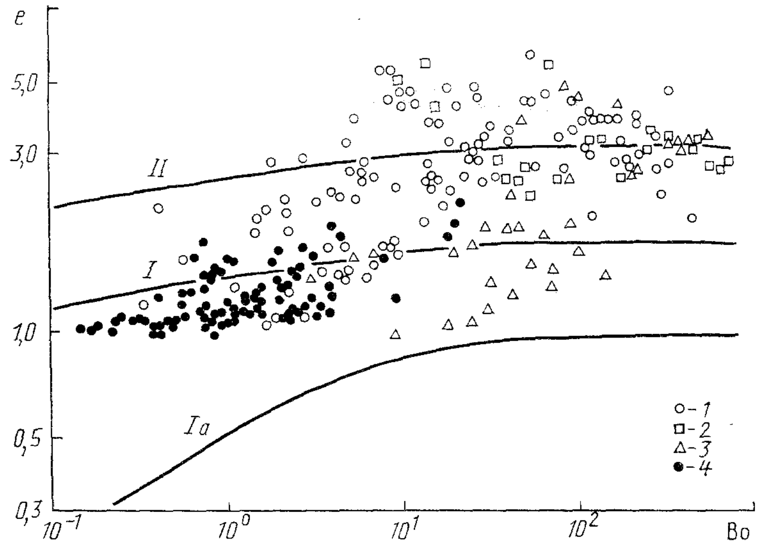

Fig. 2. Eccentricity of bubbles and drops: 1, 2, 3) bubbles in an inviscid light liquid, an inviscid heavy liquid, and in a viscous liquid; 4) drops in a liquid.

capable of increasing with time is less than the length of the frontal arc of the body, and after a dwell time of the wave equal to the distance from the forward point of the body to the rear point the wave amplitude α should increase to a value approximately equal to the wavelength, which leads to breakup of the body.

Using the expression for the minimum wavelength and the rate of growth of its amplitude [8] we can obtain the result that for a body of ellipsoidal shape the first condition is fulfilled when

$$\mathrm{Ka}_1 \geqslant 2^{20}(\rho_1+\rho_2)^6/\beta^4\rho_1^2\rho_2^4\mathrm{Bo}\,[2+\mathrm{Bo}/2]^4, \tag{9}$$

and the second condition holds when

$$\mathrm{Ka}_1 \geqslant 2^{20}/\mathrm{Bo}\,\{\beta\rho_2\rho_1^{1/2}\,[2+\mathrm{Bo}/2]/(\rho_1+\rho_2)^{3/2} - 2^2\rho_1/\pi\,(\rho_1+\rho_2)\}^4. \tag{10}$$

For a cup-shaped bubble we have, respectively,

$$\mathrm{Ka}_1 \geqslant 2^{10}\pi^6\rho_1^4\,(\cos^3\theta - 3\cos\theta + 2)^2/\beta^4\theta^6\rho_2^4\mathrm{Bo}\,[2+\mathrm{Bo}/2]^4, \tag{11}$$

$$\mathrm{Ka}_1 \geqslant 2^{16}\pi^6\,(\cos^3\theta - 3\cos\theta + 2)^2/\theta^6\mathrm{Bo}\,\{\beta\rho_2\,[2+\mathrm{Bo}/2]\,\rho_1 - 2^{4/3}\,(\cos^3\theta - 3\cos\theta + 2)^{1/3}/\theta\}^4. \tag{12}$$

The analytical expressions obtained were compared with the experimental results.

Figure 1 shows data on conditions for the start of deformation of bubbles and drops [3, 9-13] or the appearance in them of circulation flows [14-16], and also transition of bubbles from an ellipsoidal to a cup shape [1, 3, 11, 17, 18]. Figure 1 also shows curves 1 and 2 computed from Eq. (3) for k_f = 24 and 48 [3], respectively, and curve 3 computed from Eq. (4).

Analysis of the mutual location of the experimental data and the computed curves shows that on the boundaries of the region I generated by the latter, there are points corresponding to the start of transition from a sphere to an ellipsoid or the appearance of circulation flows in the bubbles and drops. Here condition (3) is governing for $\mathrm{Ka}_1 \geq 3^4 k_f{}^4/2^{15}$, and condition (4) governs for smaller values of Ka_1. In region III the gas bubbles and drops are cup-shaped. More will be written below concerning the transition regions II and IV.

Data on the eccentricity of bubbles and drops presented in [9, 10, 12, 17, 19-28] are shown in Fig. 2. Figure 2 also shows the computed lines from Eq. (7) with $u_{fr}/u_\lambda = 1$ (curve I) and $u_{fr}/u_\lambda = 0$ (curve Ia) for an ellipsoidal shape of bubbles and drops, and also the computed line II, obtained by solving Eq. (8) for θ with given values of Bo and with subsequent determination of the eccentricity of a cup-shaped bubble $e \equiv 2a_0/b_0 = 2\sin\theta/(1 - \cos\theta)$.

From analysis of Fig. 2 we note that both for drops and for bubbles ascending in an inviscid liquid, at small enough values of Bo the eccentricity is often somewhat less than as computed from Eq. (7) under the assumption that the body is ellipsoidal and the speed at the interface is the ascent speed. For viscous liquids this position holds up to larger values of Bo, the higher the viscosity of the surrounding liquids. When condition (3) is reached the ellipsoidal shape changes to cup-shaped. Here in the rear part of the body the circulation flows act in the wake to produce partial transition from a convex shape to planar, and as the size increases, and correspondingly the speed, this phenomenon becomes more pronounced. This region of the data points is located between lines I and II, and because of the complexity of the body shape it cannot be written analytically, and Eq. (5) can only be solved numerically. Finally, under the condition that the planar rearward part of an ellipsoidal body passes through its equatorial plane, the body acquires a cup shape ($\theta < \pi/2$), and the experimental data are grouped near line II, computed from Eq. (8). The higher the viscosity of surrounding liquid, θ is 45-75° [21-24], very often 55-65°, the value $\theta \to 62°$ and $e \to 3.33$. In the experiment θ is 45-75° [21-24], very often 55-65°, and does not depend on the viscosity of the surrounding liquid. For the two-dimensional experiments [21, 24] the rules for the formation of bubbles are the same as for three-dimensional, which confirms the hypothesis as to the wave nature of this process. It should be noted that the observed scatter of the experimental data obtained in the different investigations for the same conditions is associated primarily with oscillations of bubbles and drops due to the hydrodynamic processes in the wake behind the body. Their deviations from the computed values are linked with the assumptions, and in particular, that $\bar{u}_{fr} \equiv u_{fr}/u_\lambda = 1$.

For ellipsoidal bubbles and drops, according to Eq. (7), knowing the values of e and Bo, and for cup-shaped bubbles, according to Eq. (8), knowing the values of θ and Bo, one can determine u_{fr}. It turned out that in each of the experiments, for a given pair – disperse phase/continuum phase – with increase of the equivalent diameter of the disperse phase, or, which is the same thing, of the quantity Bo, the value of u_{fr} first remained more or less constant, which could be considered as conserving the constant body shape, and then increased, beginning with a certain value of Bo.

On the average, for ellipsoidal bubbles floating in an inviscid liquid $\bar{u}_{fr} \approx 0.88$, and for cup-shaped bubbles $\bar{u}_{fr} \approx 0.86$. For the motion of drops in inviscid liquids $\bar{u}_{fr} \simeq 0.76$. Thus, the hypothesis that the speed of bubbles and drops at the boundary is close to their speed of motion [6] is confirmed for an inviscid surrounding medium. According to [6], for specific conditions one must take into account the viscosity of the phases, and $\bar{u}_{fr}$ depends on the quantity $\mu \equiv \mu_2/\mu_1$. However, for bubbles, because $\mu_2 \ll \mu_1$, this influence can be neglected [6]. In fact, for bubbles floating in a viscous liquid the speed at the boundary falls off with increase of viscosity of the liquid, and is ~0.2 for the conditions of [12]. For drops moving in a viscous liquid the value of $\bar{u}_{fr}$ falls to 0.5-0.6 for the conditions of [25].

As was shown above, the computed value of u_{fr} for a specific value of Bo begins to increase noticeably, which can be considered as a transition from ellipsoidal to cup shape. In Fig. 1 these conditions are shown by points, which are located predominantly in region II. Thus, one can consider region I as the region of existence of spherical bubbles and drops moving in a liquid. When the liquid moves relative to them with a possible speed exceeding the speed of motion of the gravity wave, the wavelength is equal to their perimeter, and the appearance of relative speed in the bubbles and drops causes circulation, whose velocity in an inviscid surrounding liquid is comparable with their speed of ascent, and transition occurs from a spherical shape to a near-ellipsoidal shape. For inviscid liquids this transition occurs via region II, and for viscous liquids, evidently via region IV. At the same time, because of the presence of a wake in the rearward part of the moving body one sees the formation of a planar area, which signifies transition from an ellipsoidal shape to cup-shaped, and for an intermediate shape (an ellipsoid with a planar area in the rear part) a numerical computation must be done.

In region III (for drops, evidently also in region II) the bubbles and drops may break up. Figure 3 shows experimental data obtained in a study of this process [6, 9, 17, 29-31], and computed lines from the analytical expression (9)-(12).

For bubbles fulfillment of the first breakup condition (lines 1a, 1b) is necessary and sufficient when they float in an inviscid liquid. In viscous liquids one observes a devia-

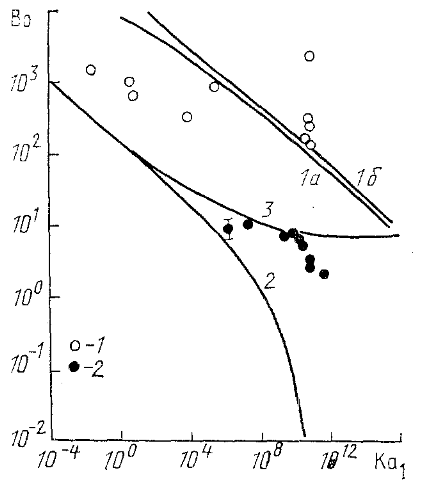

Fig. 3. Conditions for breakup of bubbles (1) and drops (2): 1a) $\lambda_{min} \leq 2R\theta$, 1b) $\lambda_{min} \leq \pi D/2$.

tion from this rule which is greater, the larger the viscosity of the liquid, due, apparently to the laws mentioned above in the development of circulation in this case. Fulfillment of the second breakup condition presupposes that in Eq. (10) the difference of the parts of the expression in curved brackets is greater than zero. For values of the ratio ρ_2/ρ_1 ordinarily met in practice this can occur for Bo > 10^4-10^5, which is considerably greater than observed. Thus, fulfillment of the first condition for inviscid liquid achieves breakup of the bubbles. This agrees with the breakup mechanism described in [17], when a toroidal volume of gas separates from the edge of a cup-shaped bubble, if the length of the frontal arc of the spherical part of the bubble surface exceeds a critical value.

Judging from Fig. 3, for bubbles it is more likely to be necessary to fulfill both the first condition (line 2) and the second condition (line 3), the second being necessary and sufficient. This also agrees with the picture of breakup described in [31], when, as a result of increase of amplitude of the ambient surface of the bubble, the wave, on reaching the condition $\alpha \approx \lambda$ in the rear part of a moving drop, forms a daughter drop smaller than the original.

A comparison of the solutions obtained with those of [3, 5, 6, 13, 20, 24, 25, 31] and referenced in the reviews [32, 33] shows that with the wave approach in specific cases one can obtain analytical relations, including an explicit form, for the eccentricity of bubbles and drops and the critical conditions for their change of shape.

CONCLUSIONS

1. Using wave theory we have obtained analytical expressions to describe the shape of bubbles and drops moving in liquids, and the conditions for their breakup.

2. The known experimental data agree quite well with values computed using the expressions derived.

NOTATION

λ, α, wavelength and wave amplitude; δ, body thickness at a given point; u_λ, u_σ, u, u_{fr}, speed of gravity and capillary waves, motion of the liquid relative to the body, including its boundary; D_e, equivalent body diameter; ρ_1, ρ_2, density of the continuum and disperse phases; μ_1, μ_2, dynamic viscosity of the continuum and disperse phases; σ, interphase tension; g, acceleration due to gravity; $\Delta\rho = |\rho_1 - \rho_2|$; c_f, drag coefficient of body shape; k_f, coefficient of proportionality; $Re \equiv uD_e\rho_1/\mu_1$, Reynolds number; $Bo \equiv \Delta\rho g D_e^2/\sigma$, Boyd number; $Ka_1 \equiv \sigma^3(\rho_1 + \rho_2)^2/\Delta\rho g\mu_1^4$, modified Kapitsa number for the continuum phase; τ, τ_0, time, duration of flotation of a body at a distance equal to its maximum thickness; a_0, semimajor axis of ellipsoid, base radius of the cup-shaped body; b_0, semiminor axis of ellipsoid, maximum thickness of cup-shaped body; θ, semiangle at vertex of cup-shaped body; a, distance from the vertical body axis; e, eccentricity.

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